Terrestrial Locomotion Modeling Bio-inspired by Elongated Animals

ECOLE DES MINES DE NANTES

IRCCyN

Shaukat ALI (PhD Student)
Ecole des Mines, Nantes, France
(shaukat.ali@mines-nantes.fr)

Frédéric BOYER (Pr.)
Ecole des Mines, Nantes, France
(frederic.boyer@mines-nantes.fr)

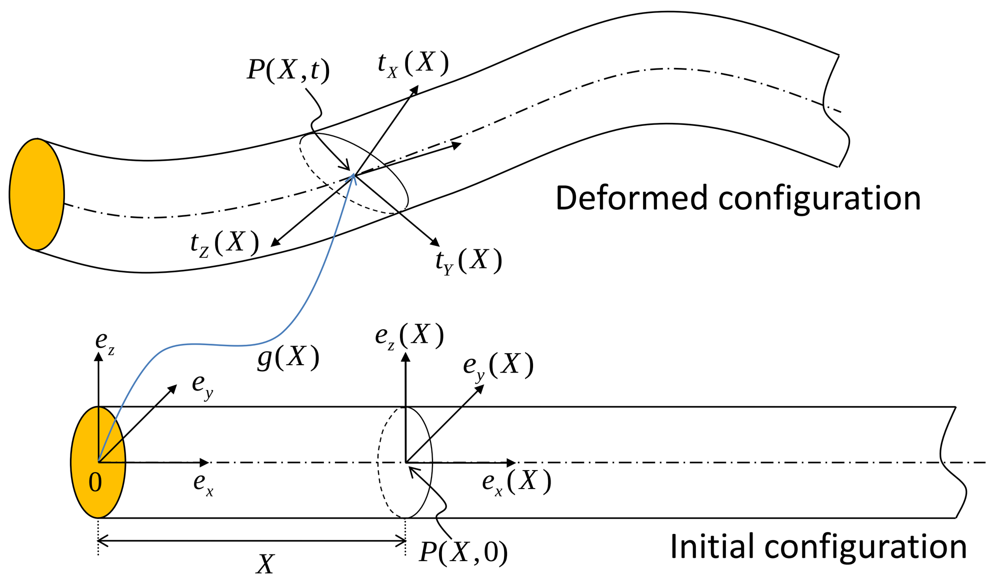
Mathieu POREZ (Dr.)
Ecole des Mines, Nantes, France
(mathieu.porez@mines-nantes.fr)

- TO PRESENT A UNIFIED DYNAMIC MODELING APPROACH FOR ***HYPER-REDUNDANT CONTINUUM SYSTEMS*** BIO-INSPIRED BY ELONGATED ANIMALS
- TO DEVELOP AN ALGORITHM CAPABLE OF COMPUTING THE ***NET MOTION*** AND INTERNAL ***CONTROL TORQUES*** OF THE HYPER-REDUNDANT CONTINUUM SYSTEMS
- APPLICATION TO ELONGATED ANIMALS: VERTEBRATES SUCH AS SNAKES, AND INVERTEBRATES SUCH AS EARTHWORMS AND INCHWORMS

GENERAL APPROACH

BEAM THEORY vs. CONTINUUM SYSTEMS

To model a Hyper-redundant Continuum System as a 3D Cosserat Beam with imposed strain fields between sections



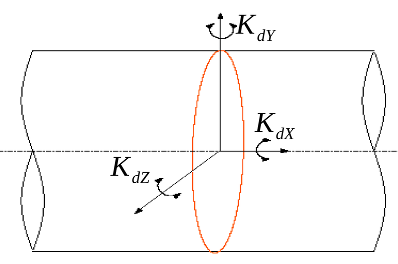
Strain fields:

$K_d(t) = (K_{dX}, K_{dY}, K_{dZ})$

Torsion | Bending

$\Gamma_d(t) = (\Gamma_{dX}, \Gamma_{dY}, \Gamma_{dZ})$

Stretching | Transverse shear



BEAM KINEMATICS

To model the inter-vertebral kinematics (or joint kinematics in case of robots) as actuated strain fields between sections of the beam

$$\hat{\xi}_d(t) = \begin{pmatrix} \hat{K}_d(t) & \Gamma_d(t) \\ 0_{3\times 3} & 0 \end{pmatrix} = g^{-1}g'$$

Case	Constraints	Degrees of Freedom (DoFs)	Beam	Remarks
1	No constraints	06	Timoshenko-Reissner	
2	$\Gamma_{dY} = \Gamma_{dZ} = 0$	04	Extensible Kirchoff	Sections stay perpendicular to vertebral axis
3	Case 2 with $\Gamma_{dX} = 1$	03	Inextensible Kirchoff	Infinitesimal version of a spherical joint
4	Case 3 with $K_{dX} = 0$	02	No corresponding beam	3D bending always produce torsion
5	Case 4 with $K_{dY} = 0$	01	Inextensible planar Kirchoff	Planar case with yaw DoF actuation

LIE GROUP THEORY vs. LOCOMOTION

Lie group of Transformations: $G \subseteq SE(3)$

Homogenous Transformation element in G: $g \in SE(3)$

Lie Algebra of G: $\mathcal{G} \subseteq se(3)$

Configuration Spaces in Locomotion

Configuration Space: as a principal fiber bundle

$$\mathcal{C}_2 = G \times \mathcal{S}$$

Configuration Space: as a functional space of curves in $SE(3)$

$$\mathcal{C}_1 = \{ g : X \in [0,l] \mapsto g(X) \in SE(3) \}$$

Configuration Space: as a functional space of curves in $se(3)$

$$\mathcal{S} = \{ \xi : X \in [0,l] \mapsto \xi(X) \in se(3) \}$$

Vector fields in $se(3)$

Time-twist field: $\hat{\eta} : X \in [0,l] \mapsto \hat{\eta}(X,t) = g^{-1}\dot{g} \in se(3)$

Space-twist field: $\hat{\xi} : X \in [0,l] \mapsto \hat{\xi}(X,t) = g^{-1}g' \in se(3)$

MATHEMATICAL MODELS + ALGORITHM

MODELS OF SYSTEM

Model of transformations: $g' = g\hat{\xi}(t)$

Boundary Condition: $g(X=0) = g_o$

Model of Velocities: $\eta' = -ad_{\xi_d}(\eta) + \dot{\xi}_d(t)$

Boundary Condition: $\eta(X=0) = \eta_o$

Model of Accelerations: $\dot{\eta}' = -ad_{\xi_d}(\dot{\eta}) - ad_{\dot{\xi}_d}(\eta) + \ddot{\xi}_d(t)$

Boundary Condition: $\dot{\eta}(X=0) = \dot{\eta}_o$

Model of Internal Dynamics

$$\mathcal{M}\dot{\eta} - ad_{\eta}^{*}(\mathcal{M}\eta) - \Lambda' + ad_{\xi}^{*}(\Lambda) = \bar{F}$$

Boundary Conditions: $\Lambda(X=0) = -F_{-}$, $\Lambda(X=l) = F_{+}$

Internal dynamics are defined on $\mathcal{C}_1$

Model of External Dynamics

$$\begin{pmatrix} \dot{\eta}_o \\ \dot{g}_o \end{pmatrix} = \begin{pmatrix} \mathcal{M}_o^{-1} F_o \\ g_o \hat{\eta}_o \end{pmatrix}$$

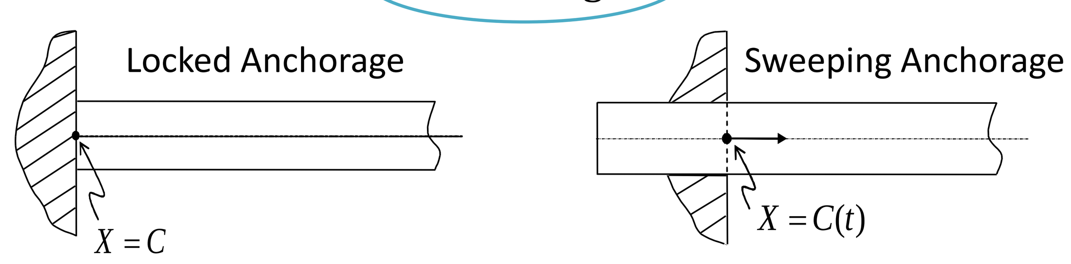
External dynamics are defined on $\mathcal{C}_2$

CONTACT

The nature of contact between system and surrounding plays an important role in defining the mode of locomotion

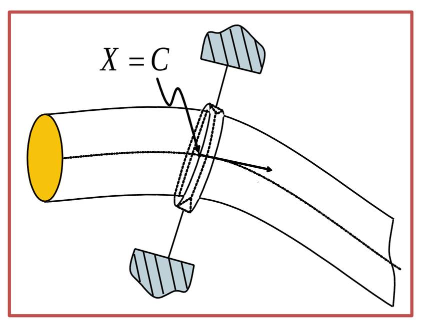
- The contacts are assumed ideal

Anchorage



Anchorages are modeled as ***Bilateral Holonomic Constraints***

Annular Contact



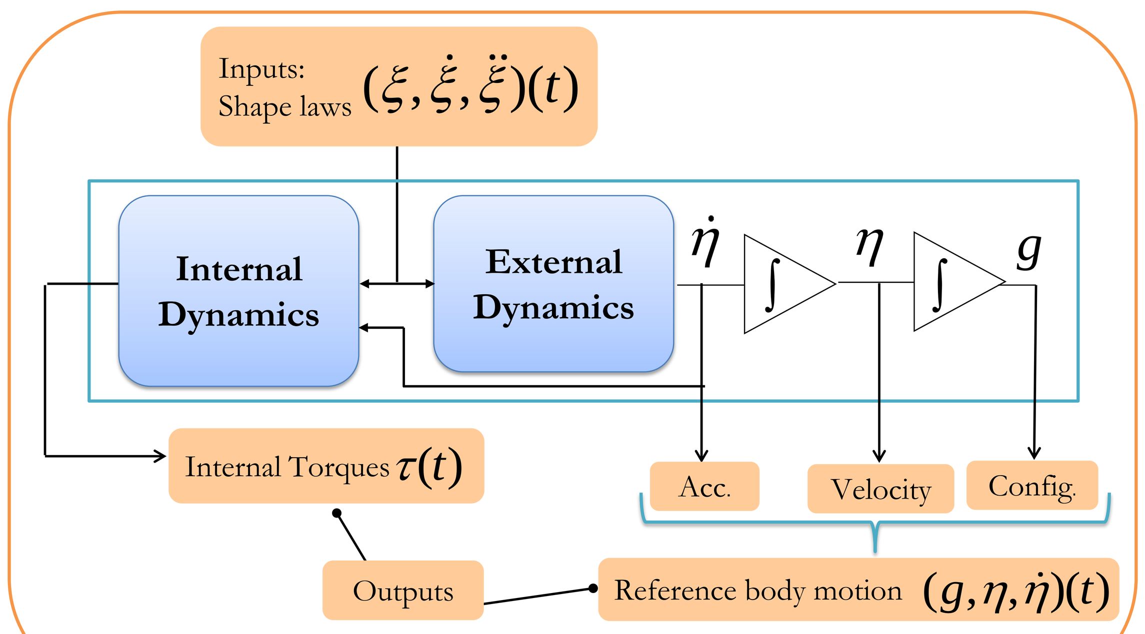
Annular contacts are modeled as ***Non-holonomic Constraints***

The contact forces are identified as Lagrange Multipliers associated to the constraints

ALGORITHM

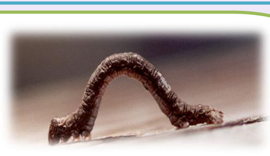
The proposed algorithm combines all these approaches and models to give a unified framework for efficient computation of:

- The net motion of the reference body of continuum system
- The torques required to impose the desired strain law

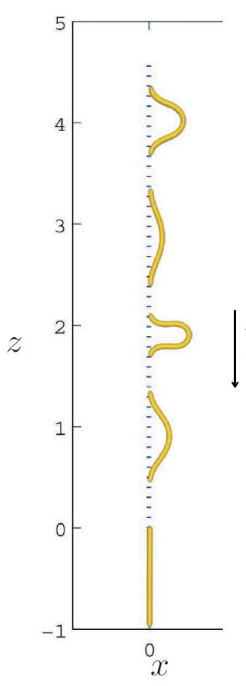


ILLUSTRATIVE EXAMPLES + SIMULATION RESULTS

INCHWORM

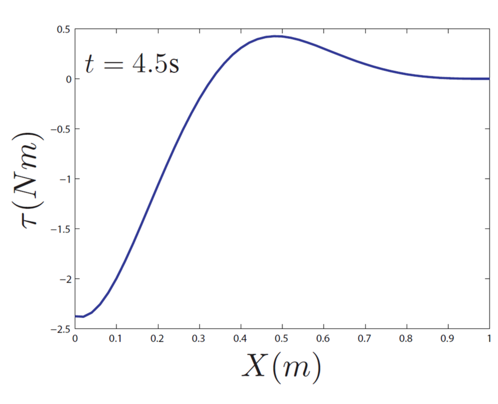


Group	$G = SE(2)$
Type of beam	2D actuated Kirchhoff beam
Type of contact	Locked Anchorage
Shape law	$K_{dY}(X,t) = \alpha \sin^2(\omega t)\left(\frac{2\pi}{l}\right)\cos\left(\frac{2\pi}{l}(X-l)\right)$



Climbing motion of an Inchworm

The locked anchorage is periodically changed between head and tail

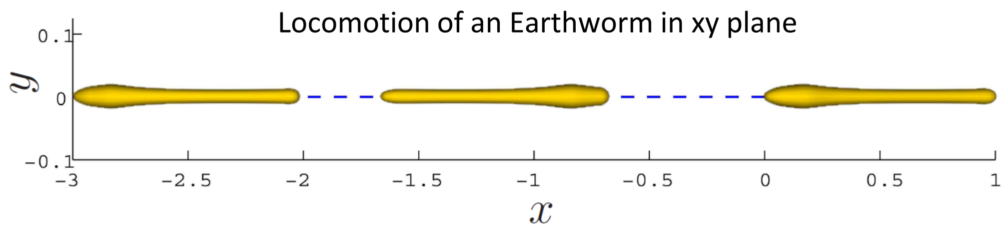


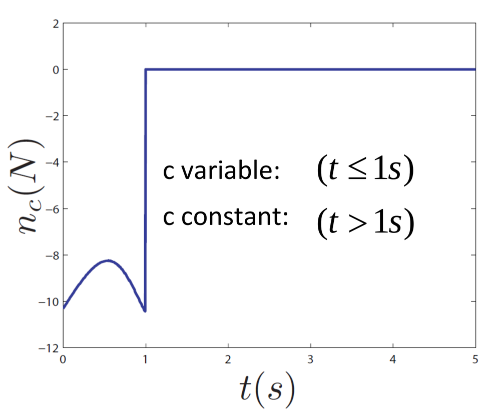
Torque distribution over the length

EARTHWORM

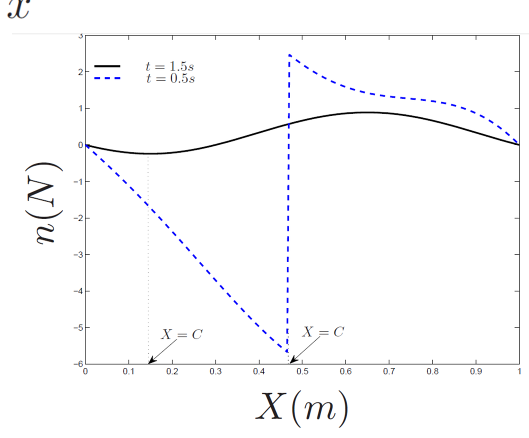
Group	$G = SE(1)$
Type of beam	1D actuated extensible Kirchhoff beam
Type of contact	Sweeping Anchorage
Connection	$\dot{x}_o = -\Gamma_{dX}(C_i)\dot{C}_i(t) - \int_0^{C_i(t)} \dot{\Gamma}_{dX}\, dX$
Shape law	$\Gamma_{dX}(X,t) = 1 + \varepsilon \sin(\omega(t - X/c))$

Locomotion of an Earthworm in xy plane



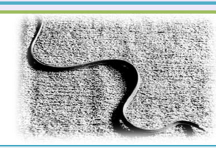


Contact forces

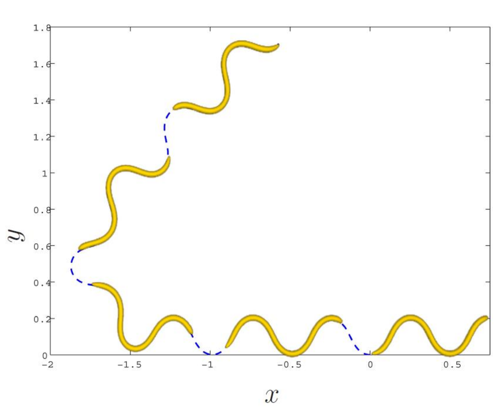


Force distribution over the length

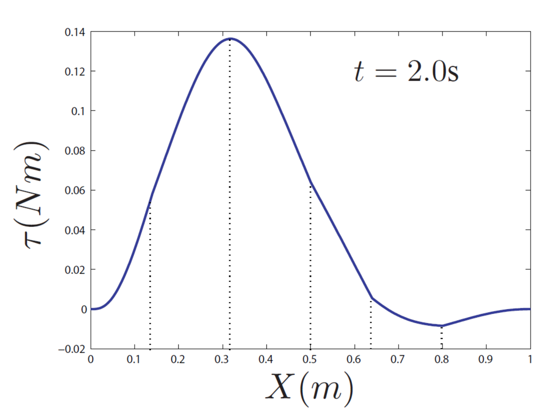
SNAKE



Group	$G = SE(2)$
Type of beam	2D actuated Kirchhoff beam
Type of contact	Annular
Connection	$\eta_o = \begin{pmatrix} 1/K_o' \\ K_o/K_o' \end{pmatrix} \dot{K}_o$
Shape law	$K_{dZ}(X,t) = f\left(X + \int_o^t V_o(\tau)d\tau\right)$
Compatibility	$\dot{K}_{dZ} = V_o K_{dZ}'$



Turning locomotion in xy plane



Torque distribution over the length