From Fermat's Principle to Invisibility

Janos Perczel and Ulf Leonhardt

University of St Andrews

EPSRC
Engineering and Physical Sciences Research Council

Introduction:

• By Fermat`s Principle, light follows the "quickest" optical path (**Fig. 1 & 2**).

• By making the quickest optical paths curved, the trajectory of the light rays can be turned into a loop (**Fig. 3, 4 & 5**).

• This property of light can be used to produce striking optical phenomena, such as invisibility.

Summary:

• We implemented the Invisible Sphere and Maxwell's Fisheye on two virtual sheets, and connected these sheets by a "branch cut," through which light can pass from one medium to another (**Fig. 11**).

• We transformed these two virtual sheets into a single sheet, corresponding to physical space (**Fig. 12**).

• The area inaccessible to light on the lower virtual sheet transforms to an invisible chamber – thus an invisibility device was constructed.

Fundamental ideas for Invisibility

I. Fermat`s Principle:

• Light follows the quickest path in optical media (glass, water...). This path depends on the optical properties of the media the light is passing through.

• Therefore, a sudden change in medium implies a sudden change in the light trajectory.

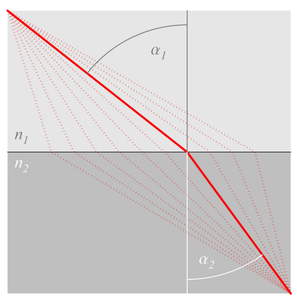

Fig. 1. Light follows the quickest optical path

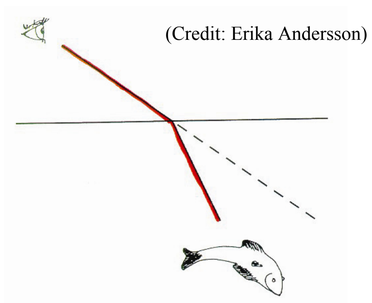

Fig. 2. Change in media changes the quickest optical path

• If the medium varies gradually, the quickest path will be smoothly curved. This is the basis of mirages and of the two special media introduced in the next section:

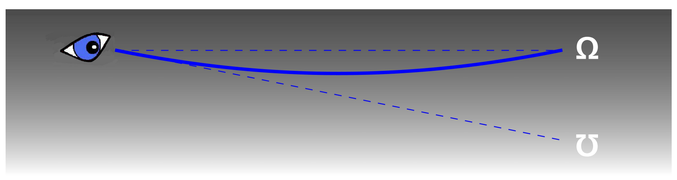

Fig. 3. Smooth change of medium can produce a mirage

Conclusion: Optical media change our perception of space.

II. The Invisible Sphere and Maxwell's Fisheye:

The **Invisible Sphere** and **Maxwell's Fisheye** are special optical media in which Fermat's Principle gives rise to light trajectories with loops.

• Light rays entering the Invisible Sphere (Fig. 4) emerge without their original direction altered, thus the Sphere becomes *invisible*.

• The Invisible Sphere on its own cannot make anything invisible, since its entire volume is accessible to light.

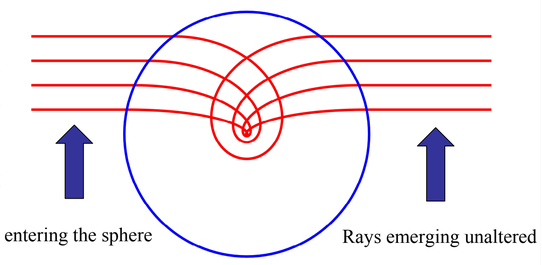

Figure 4. The Invisible Sphere

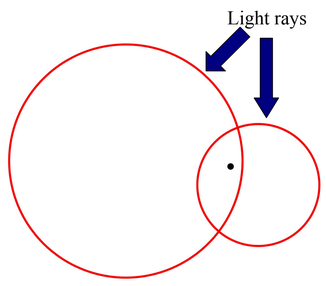

Fig. 5. Circular loops in the Fisheye

• In Maxwell's Fisheye, light goes around in circles (Fig. 5).

• When a spherical mirror is introduced, the loops are still closed (Fig. 6).

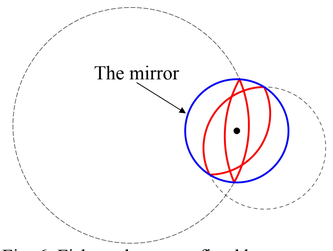

Fig. 6. Fisheye-loops confined by a mirror

Conclusion: The Invisible Sphere and Maxwell's Fisheye make light propagate in loops.

III. Expansion of space through coordinate transformation:

• By the transformation of spatial coordinates a small region of space can be expanded into a much larger – even infinite – hidden region.

This is the scientific basis of Dr Who's Tardis:

Source: doctorwho1.com

Source: brusimm.com

• Below, we transformed a region of physical space (red region in Fig. 9) to a hidden virtual plane (red lower plane in Fig. 10 – see black arrow). We matched the colours to show which region maps to which.

• In both figures, light rays can cross from the blue to the red regions through the yellow branch cut (Fig. 9 & 10).

• In Fig. 10, the upper sheet is visible and the Underworld – although hidden – is accessible through the yellow branch cut; just as the hidden inside of Dr Who's Tardis is accessible through its door.

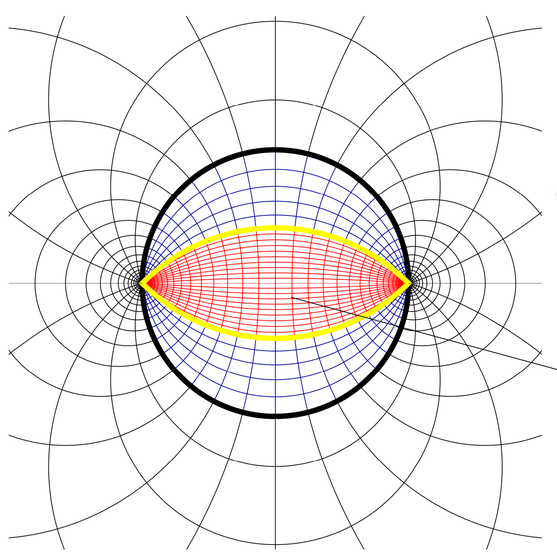

Fig. 9. Physical space

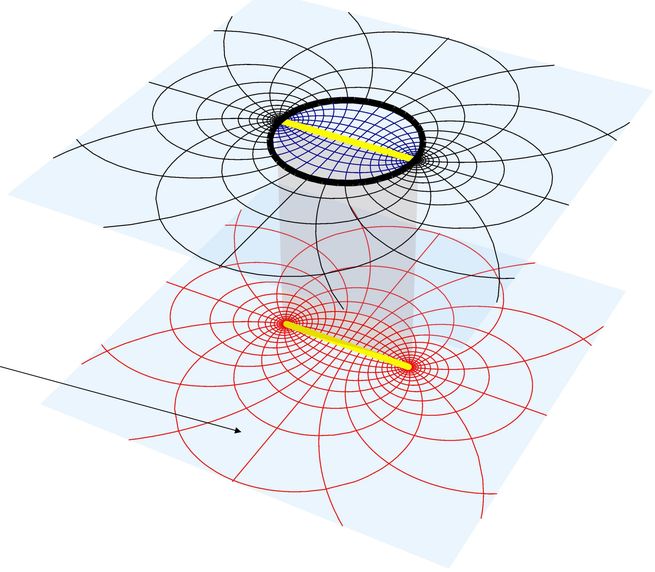

Fig. 10. Expanded virtual space

Conclusion: A finite region of physical space can be expanded into a hidden virtual space.

An Invisibility Device

We can now put the above ideas together to construct an invisibility device!

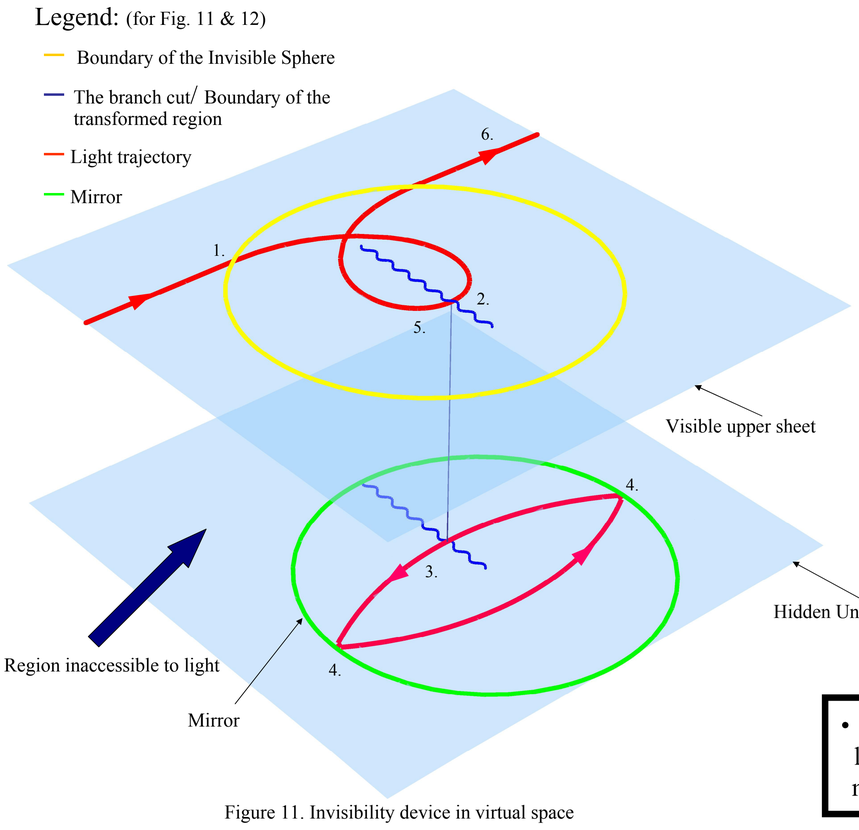

Figure 11. Invisibility device in virtual space

• On the visible upper sheet of Fig. 11, we implement the Invisible Sphere, and in the hidden Underworld we place Maxwell's Fisheye with the mirror.

• In Fig. 12. we reverse the coordinate transformations and obtain the trajectories and the position of the mirror in physical space. This physical medium can be engineered into metamaterials.

• The numbers in Figs. 11. & 12. explain the path of light in virtual and physical space respectively. The light ray…

1) … enters the Invisible Sphere
2) … crosses the branch cut
3) … starts journey in the Underworld
4) … bounces off the mirror
5) … emerges from the Underworld
6) … continues its journey as if nothing had happened

• In Fig. 11. the Underworld outside the green mirror is inaccessible to light. It transforms to a physical region bounded by a spindle-shaped mirror in Fig. 12. **Objects placed in this region are made *invisible!***

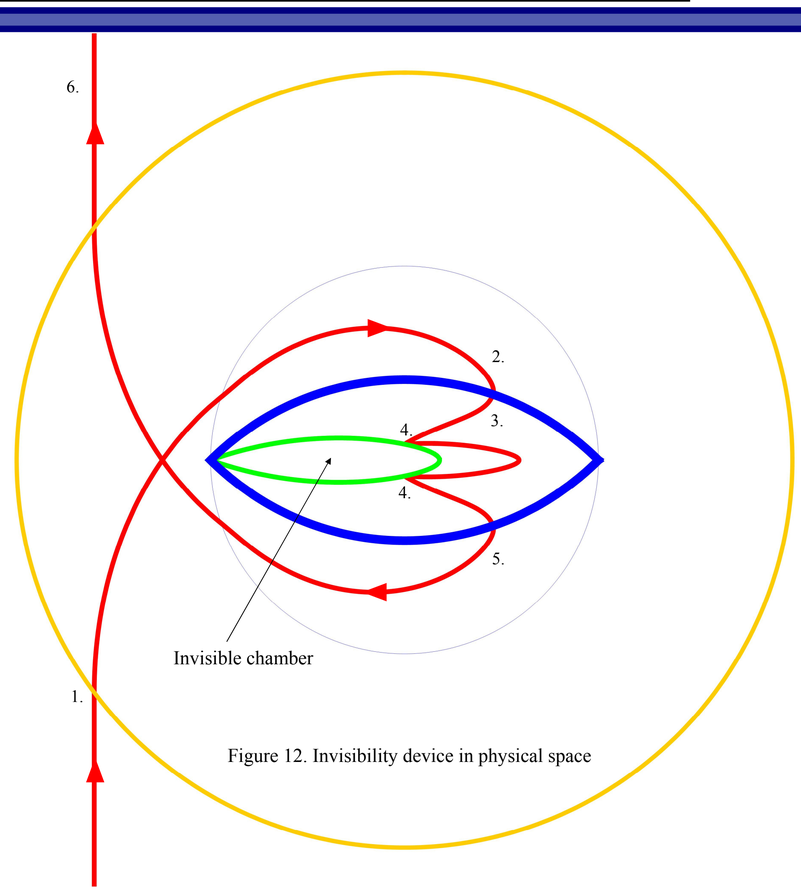

Figure 12. Invisibility device in physical space

References:

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• J. B. Pendry, D. Schurig, D. R. Smith, *Contolling Electromagnetic Fields,* Science **312,** 1780-1782 (2006);
• U. Leonhardt and T. Tyc, Broadband Invisibility by Non-Euclidean Cloaking, Science **323**, 110-112 (2009);

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